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# THE MATHEMATICS TEACHER

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## TERMS AND SYMBOLS IN ELEMENTARY MATHEMATICS.

A PRELIMINARY REPORT BY

THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.\*

**A. Limitations Imposed by the Committee upon its Work.**—The Committee feels that it should place upon itself certain general limitations, as follows:

1. No attempt should be made to impose the phraseology of any definition upon the schools, although the Committee should state clearly its general views as to the meaning of disputed terms.
2. No effort should be made to change any well-defined current usage unless there is a strong reason for doing so, which reason is supported by the best authority.
3. So far as reasonable, *the terms used should be international*. This principle excludes the use of all individual efforts at coining new terms except under circumstances of great urgency. The individual opinions of the members, as indeed of any teacher or body of teachers, should have little weight in comparison with international usage if this usage is definite. If an idea has to be expressed so often in elementary mathematics that it becomes necessary to invent a single term or symbol for the purpose, this invention is necessarily the work of an individual; but it is highly desirable, even in this case,

\* The first draft of this report was prepared by a subcommittee consisting of David Eugene Smith (chairman), W. W. Hart, H. E. Hawkes, E. R. Hedrick and H. E. Slaught. It was revised by the National Committee at its meeting December 29 and 30, 1920, and is now issued as one of its preliminary reports.

that it should receive the sanction of high authority before it is used in any system of examinations.

4. On account of the large number of terms and symbols now in use, the recommendations to be made should be typical rather than exhaustive.

### I. GEOMETRY.

**B. Undefined Terms.**—The Committee recommends that little attempt be made to define, with any approach to precision, terms whose definitions are not needed as parts of a proof.

Especially is it recommended, that no attempt be made to define completely such terms as *space*, *magnitude*, *point*, *straight line*, *surface*, *plane*, *direction*, *distance*, and *solid*, although the significance of such terms should be made clear by informal explanations and discussions.

**C. Definite Usage Recommended.**—While not seeking to dictate the phraseology of any definition, it is the opinion of the Committee that the following general usage is desirable:

1. *Circle* should be considered as the curve; but where no ambiguity arises, the word "circle" may be used to refer either to the curve or to the part of the plane inclosed by it.

2. *Polygon* (including *triangle*, *square*, *parallelogram*, and the like) should be considered, by analogy to a circle, as a closed broken line; but where no ambiguity arises, the word polygon may be used to refer either to the broken line or to the part of the plane inclosed by it.

3. *Area of a circle* should be defined as the area (numerical measure) of the portion of the plane inclosed by the circle. Similarly, *segment of a circle* should be defined as that portion of the plane bounded by a chord and its arc. *Area of a polygon* should be treated in the same way.

4. *Solids.* The usage above recommended with respect to plane figures is also recommended with respect to solids. For example, *sphere* should be regarded as a surface, its volume should be defined in a manner similar to the area of a circle, and the double use of the word should be allowed where no ambiguity arises. A similar usage should obtain with respect to such terms as *polyhedron*, *cone*, and *cylinder*.

5. *Circumference* should be considered as the length

(numerical measure) of the circle (line). Similarly, *perimeter* should be defined as the length of the broken line which forms a polygon, that is, as the sum of the lengths of the sides.

6. *Obtuse angle* should be defined as an angle greater than a right angle and less than a straight angle, and should therefore not be defined merely as an angle greater than a right angle.

7. The term *right triangle* should be preferred to "right-angled triangle," this usage being now so standardized in this country that it may properly be continued in spite of the fact that it is not international. Similarly for *acute triangle*, *obtuse triangle*, and *oblique triangle*.

8. Such English plurals as *formulas*, and *polyhedrons*, should be used in place of the Latin and Greek plurals. Such unnecessary Latin abbreviations as *Q.E.D.* and *Q.E.F.* should be dropped.

9. The definitions of *axiom* and *postulate* vary so much that the Committee does not undertake to distinguish between them.

**D. Terms Made General.**—It is the recommendation of the Committee that the modern tendency of having terms made as general as possible should be followed. For example:

1. *Isosceles triangle* should be defined as a triangle having two equal sides. There should be no limitation to two and only two equal sides.

2. *Rectangle* should be considered as including a square as a special case.

3. *Parallelogram* should be considered as including a rectangle, and hence a square, as a special case.

4. *Segment* should be used to designate the part of a straight line included between two of its points as well as the part of the plane bounded by an arc of a circle and its chord, this being the usage generally recognized by modern writers.

**E. Terms to Be Abandoned.**—It is the opinion of the Committee that the following terms are not of enough consequence in elementary mathematics at the present time to make their recognition desirable in examinations and that they serve chiefly to increase the technical vocabulary to the point of being burdensome and unnecessary:

1. *Antecedent* and *consequent*.

2. *Third proportional* and *fourth proportional*.

3. *Equivalent.* An unnecessary substitute for the more precise expressions "equal in area" and "equal in volume," or (where no confusion is likely to arise) for the single word "equal."

4. *Trapezium.*

5. *Scholium, lemma, oblong, scalene triangle, sect, perigon, rhomboid* (the term "oblique parallelogram" being sufficient), and *reflex angle* (in elementary geometry).

6. Terms like *flat angle, whole angle, and conjugate angle* are not of enough value in an elementary course to make it desirable to recommend them.

7. *Subtend*, a word which has no longer any etymological meaning to most students and teachers of geometry. While its use will naturally continue for some time to come, teachers may safely incline to such forms as the following: "In the same circle equal arcs *have equal chords.*"

8. *Homologous*, the less technical term "corresponding" being preferable.

9. Guided by principle A3 and its interpretation, the Committee advises against the use of such terms as the following: *Angle-bisector, angle-sum, consecutive interior angles, supplementary consecutive-exterior angles, quader* (for rectangular solid), *sect, explement, transverse angles.*

10. It is unfortunate that it still seems to be necessary to use such a term as *parallelepiped*, but we seem to have no satisfactory substitute. For rectangular parallelepiped, however, the use of *rectangular solid* is recommended. If the terms were more generally used in elementary geometry it would be desirable to consider carefully whether better ones could not be found for the purposes than *isoperimetric, apothem, icosahedron, and dodecahedron.*

**F. Symbols in Elementary Geometry.**—It should be recognized that a symbol like  $\perp$  is merely a piece of shorthand designed to abbreviate and to allow for an easy grasp of a written or printed statement. Many teachers and a few writers make an extreme use of symbols, coining new ones to meet their own views as to usefulness, and this practice is naturally open to

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objection.\* There are, however, certain symbols that are international, and certain others of which the meaning is at once apparent and which are sufficiently useful and generally enough recognized to be recommended.

For example, the symbols for triangle,  $\Delta$ , and circle,  $\circ$ , are international, although used more extensively in the United States than in other countries. Their use, with their customary plurals is recommended.

The symbol  $\perp$ , generally read as representing the single word "perpendicular" but sometimes as standing for the phrase "is perpendicular to," is fairly international and the meaning is apparent. Its use is therefore recommended. On account of such a phrase as "the  $\perp AB$ ," the first of the above readings is likely to be the more widely used, but in either case there is no chance for confusion.

The symbol  $\parallel$  for "parallel" or "is parallel to" is fairly international and is recommended.

The symbol  $\sim$  for "similer" or "is similar to" is international and is recommended.

The symbols  $\cong$  and  $\equiv$  for "congruent" or "is congruent to" both have a considerable use in this country. The Committee feels that the former, which is fairly international, is to be preferred because it is the more distinctive and suggestive.

The symbol  $\angle$  for angle is, because of its simplicity, coming to be generally preferred to any other and is therefore recommended.

Since the following terms are not used frequently enough to render special symbols of any particular value, the world has not developed any that have general acceptance, and there seems to be no necessity for making the attempt: Square, rectangle, parallelogram, trapezoid, quadrilateral, semicircle.

The symbol  $\widehat{AB}$  for arc  $AB$  cannot be called international. While the value of the symbol  $\sim$  in place of the short word *arc* is doubtful, the Committee sees no objection to its use.

The symbol  $\therefore$  (therefore) has a value that is generally

\* This is not intended to discourage the use of algebraic methods in the solution of such geometric problems as lend themselves readily to algebraic treatment.

recognized, but the symbol . (since) is used so seldom that it should be abandoned.

With respect to the lettering of figures, the Committee calls attention for purposes of general information, to a convenient method, found in certain European and in some American textbooks, of lettering triangles: Capitals represent the vertices, corresponding small letters represent opposite sides, corresponding small Greek letters represent angles, and the primed letters represent the corresponding parts of a congruent triangle. This permits of speaking of  $\alpha$  (alpha) instead of "angle  $A$ ," and of "small  $a$ " instead of  $BC$ . The plan is by no means international with respect to the Greek letters. The Committee is prepared, however, to recommend it with the optional use of the Greek forms.

In general, it is recommended that a single letter be used to designate any geometrical magnitude, whenever there is no danger of ambiguity. The use of numbers to designate angles, such as  $\angle 1$ ,  $\angle 2$ , etc., should, however, be abandoned.

With respect to the symbolism for limits, the Committee calls attention to the fact that the symbol  $\underline{=}$  is a local one, the symbol  $\rightarrow$  (for "tends to") being both international and expressive, and having constantly grown in favor in recent years. Although the subject of limits is not generally treated scientifically in the secondary school, the idea is mentioned in geometry and a symbol may occasionally be needed.

While the teacher should be allowed freedom in the matter, the Committee feels that it is desirable to discourage the use of such purely local symbols as the following:

$\stackrel{\circ}{=}$  for "equal in degrees,"

*ass* for "two sides and an angle adjacent to one of them,"  
and

*sas* for "two sides and the included angle."

**G. Terms not Standardized.**—At the present time, there is not sufficient agreement upon which to base recommendations as to the use of the term "ray" and as to the value of terms like "coplanar," "collinear" and "concurrent" in elementary work. Many terms, similar to these, will gradually become standardized or else will naturally drop out of use.

## II. ALGEBRA AND ARITHMETIC.

**H. Terms in Algebra.**—1. With respect to equations the Committee calls attention to the fact that the classification according to degree is comparatively recent and that this probably accounts for the fact that the terminology is so unsettled. The Anglo-American custom of designating an equation of the first degree as a *simple equation* has never been satisfactory, because the term has no real significance. The most nearly international terms are *equation of the first degree* (or "first degree equation") and *linear equation*. The latter is so brief and suggestive that it should be generally adopted.

2. The term *quadratic equation* (for which the longer term "second degree equation" is an unnecessary synonym, although occasionally a convenient one) is well established. The terms *pure quadratic* and *affected quadratic* signify nothing to the pupil except as he learns the meaning from a book and the Committee recommends that they be dropped. Terms more nearly in general use are *complete quadratic* and *incomplete quadratic*. The Committee feels, however, that the distinction thus denoted is not of much importance and believes that it can well be dispensed with in elementary instruction.

3. As to other special terms, the Committee recommends abandoning, so far as possible, the use of the following: *aggregation* for grouping; *vinculum* for bar; *evolution* for finding roots, as a general topic; *involution* for finding powers; *extract* for find (a root); *absolute term* for constant term; *multiply an equation, clear of fractions, cancel and transpose*, at least until the significance of the terms is entirely clear; *aliquot part* (except in commercial work).

4. The Committee also advises the use of either *system of equations* or *set of equations* instead of "simultaneous equations," in such an expression as "solve the following set of equations," in view of the fact that at present no well established definite meaning attaches to the term "simultaneous."

5. The term *simplify* should not be used in cases where there is possibility of misunderstanding. For purposes of computation, for example, the form  $\sqrt{8}$  may be simpler than the form  $2\sqrt{2}$ , and in some cases it may be better to express  $\sqrt{\frac{3}{4}}$  as  $\sqrt{0.75}$  instead of  $\frac{1}{2}\sqrt{3}$ . In such cases, it is better to give more

explicit instructions than to use the misleading term "simplify."

6. The Committee regrets the general uncertainty in the use of the word *surd*, but it sees no reasonable chance at present of replacing it by a more definite term. It recognizes the difficulty generally met by young pupils in distinguishing between *coefficient* and *exponent*, but it feels that it is undesirable to attempt to change terms which have come to have a standardized meaning and which are reasonably simple. These considerations will probably lead to the retention of such terms as *rationalize*, *extraneous root*, *characteristic* and *mantissa*, although in the case of the last two terms, "integral part" and "fractional part" (of a logarithm) would seem to be desirable substitutes.

7. While recognizing the motive that has prompted a few teachers to speak of "positive  $x$ " instead of "plus  $x$ ," and "negative  $y$ " instead of "minus  $y$ ," the Committee feels that attempts to change general usage should not be made when based upon trivial grounds and when not sanctioned by mathematicians generally.

**I. Symbols in Algebra.**—The symbols in elementary algebra are now so well standardized as to require but few comments in a report of this kind. The Committee feels that it is desirable, however, to call attention to the following details:

1. Owing to the frequent use of the letter  $x$ , it is preferable to use the center dot (a raised period) for multiplication in the few cases in which any symbol is necessary. For example, in a case like  $1 \cdot 2 \cdot 3 \cdots (x - 1) \cdot x$ , the center dot is preferable to the symbol  $\times$ ; but in cases like  $2a(x - a)$  no symbol is necessary. The Committee recognizes that the period (as in  $a.b$ ) is more nearly international than the center dot (as in  $a.b$ ); but inasmuch as the period will continue to be used in this country as a decimal point, it is likely to cause confusion, to elementary pupils at least, to attempt to use it as a symbol for multiplication.

2. With respect to division, the symbol  $\div$  is purely Anglo-American, the symbol : serving in most countries for division as well as ratio. Since neither symbol plays any part in business life, it seems proper to consider only the needs of algebra,

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and to make more use of the fractional form and (where the meaning is clear) of the symbol  $/$ , and to drop the symbol  $\div$  in writing algebraic expressions.

3. With respect to the distinction between the use of  $+$  and  $-$  as symbols of operation and as symbols of direction, the Committee sees no reason for attempting to use smaller signs for the latter purpose, such an attempt never having received international recognition, and the need of two sets of symbols not being sufficient to warrant violating international usage and to make it necessary to burden the pupil with this additional symbolism.

4. With respect to the distinction between the symbols  $\equiv$  and  $=$  as representing respectively identity and equality, the Committee calls attention to the fact that, while the distinction is generally recognized, the consistent use of the symbols is rarely seen in practice. The Committee recommends that the symbol  $\equiv$  be not employed in examinations for the purpose of indicating identity. The teacher, however, should use both symbols if desired.

5. With respect to the root sign,  $\sqrt{\phantom{x}}$ , the Committee recognizes that convenience of writing assures its continued use in many cases instead of the fractional exponent. It is recommended, however, that in algebraic work involving complicated cases the fractional exponent be preferred. Attention is also called to the fact that the convention is quite generally accepted that the symbol  $\sqrt{a}$  ( $a$  representing a positive number) means only the positive square root and that the symbol  $\sqrt[n]{a}$  means only the principal  $n$ th root, and similarly for  $a^{1/2}$ ,  $a^{1/n}$ . The reason for this convention is apparent when we come to consider the value of  $\sqrt{4} + \sqrt{9} + \sqrt{16} + \sqrt{25}$ . This convention being agreed to, it is improper to write  $x = \sqrt{4}$ , as the complete solution of  $x^2 - 4 = 0$ , but the result should appear as  $x = \pm \sqrt{4}$ . Similarly, it is not in accord with the convention to write  $\sqrt{4} = \pm 2$ , the conventional form being  $\pm \sqrt{4} = \pm 2$ ; and for the same reason it is impossible to have  $\sqrt{(-1)^2} = -1$ , since the symbol refers only to a positive root. These distinctions are not matters to be settled by the individual opinion of a teacher or a local group of teachers; they are purely matters of convention and the Committee simply sets forth, for the

benefit of the teachers, this statement as to what the convention seems to be among the leading writers of the world at the present time.

When imaginaries are used, the symbol  $i$  should be employed instead of  $\sqrt{-1}$  except possibly in the first presentation of the subject.

7. As to the factorial symbols  $5!$  and  $\underline{5}$ , to represent  $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ , the tendency is very general to abandon the second one, probably on account of the difficulty of printing it, and the committee so recommends. This question is not, however, of much importance in the general courses in the high school.

8. With respect to symbols for an unknown quantity there has been a noteworthy change within a few years. While the Cartesian use of  $x$  and  $y$  will doubtless continue for two general unknowns, the recognition that the formula is, in the broad use of the term, the central feature of algebra has led to the extended use of the initial letter. This is simply illustrated in the direction to solve for  $r$  the equation  $A = \pi r^2$ . This custom is now international and should be fully recognized in the schools.

9. The Committee advises abandoning the double colon (:) in proportion, and the symbol  $\propto$  in variation, both of these symbols being now practically obsolete.

**J. Terms and Symbols in Arithmetic.**—1. While it is rarely wise to attempt to abandon suddenly the use of words that are well established in our language, the Committee feels called upon to express regret that we still require very young pupils, often in the primary grades, to use such terms as *subtrahend*, *addend*, *minuend* and *multiplicand*. Teachers themselves rarely understand the real significance of these words, nor do they recognize that they are comparatively modern additions to what used to be a much simpler vocabulary in arithmetic. The Committee recommends that such terms be used, if at all, only after the sixth grade.

2. Owing to the uncertainty attached to such expressions as "to three decimal places," "to thousandths," "correct to three decimal places," "correct to the nearest thousandth," the following usage is recommended: When used to specify accuracy in computation, the four expressions should be regarded as identical. The expression "to three decimal places" or "to

"thousandths" may be used in giving directions as to the extent of a computation. It then refers to a result carried only to thousandths, without considering the figure of ten-thousandths; but it should be avoided as far as possible because it is open to misunderstanding. As to the term, "significant figure," it should be noted that 0 is always significant except when used before a decimal fraction to indicate the absence of integers or, in general, when used merely to locate the decimal point. For example, the zeros underscored in the following are "significant," while the others are not: 0.5, 9.50, 102, 30,200. Further, the number 2396, if expressed correct to three significant figures, would be written 2400. It should be noted that the context or the way in which a number has been obtained is sometimes the determining factor as to the significance of a 0.

3. The pupil in arithmetic needs to see the work in the form in which he will use it in practical life outside the schoolroom. His visualization of the process should therefore not include such symbols as +, —, ×, ÷, which are helpful only in writing out the analysis of a problem or in the printed statement of the operation to be performed. Because of these facts the Committee recommends that only slight use be made of these symbols in the written work of the pupil, except in the analysis of problems. It recognizes, however, the value of such symbols in printed directions and in these analyses.

### III. GENERAL OBSERVATIONS AND RECOMMENDATIONS.

**K. General Observations.**—The Committee desires also to record its belief in two or three general observations.

1. It is very desirable to bring mathematical writing into closer touch with good usage in English writing in general. That we have failed in this particular has been the subject of frequent comment by teachers of mathematics as well as by teachers of English. This is all the more unfortunate because mathematics may be and should be a genuine help towards the acquisition of good habits in the speaking and writing of English. Under present conditions, with a style that is often stilted and in which undue compression is evident, we do not offer to the student the good models of English writing of which mathematics is capable nor, indeed, do we always offer good models of thought processes. It is to be feared that man-

teachers encourage the use of a kind of vulgar mathematical slang when they allow such words as "tan" and "cos," for tangent and cosine, and habitually call their subject by the title "math."

2. In the same general spirit the Committee wishes to observe that teachers of mathematics and writers of textbooks often seem to have gone to an extreme in searching for technical terms and for new symbols. The Committee expresses the hope that mathematics may retain as far as possible a literary flavor. It seems perfectly feasible that a printed discussion should strike the pupil as an expression of reasonable ideas in terms of reasonable English forms of expression. The fewer technical terms we introduce, the less is the subject likely to give the impression of being difficult and a mere juggling of words and symbols.

3. While recognizing the claims of euphony, the fact that a word like "historic" has a different meaning from "historical" and that confusion may occasionally arise if "arithmetic" is used as an adjective with a different pronunciation from the noun, the Committee advises that such forms as *geometric* be preferred to *geometrical*. This is already done in such terms as *analytic geometry* and *elliptic functions*, and it seems proper to extend the custom to include *arithmetic*, *geometric*, *graphic* and the like.

**L. General Recommendations**—In view of the desirability of a simplification of terms used in elementary instruction, and of establishing international usage so far as reasonable, the Committee recommends that the subject of this report be considered by a committee to be appointed by Section IV of the next International Congress of Mathematicians, such committee to contain representatives of at least the recognized international languages admitted to the meetings.

2. The Committee suggests that examining bodies, contributors to mathematical journals and authors of textbooks endeavor to follow the general principles formulated in this report.

Criticisms, comments and suggestions for the revision of this preliminary report are invited. They should be sent to J. W. Young, chairman, Hanover, New Hampshire.